ASSIGNMENT 2
UNIFORM DISTRIBUTION THEORY 2021
DUE DATE: APRIL 7, 2021

Exercise 1. ~~Let $b \geq 2$. Show that α is normal to base b if and only if α is simply normal to base b^k for all $k \geq 1$.~~

Exercise 2. Let $b \geq 2$. Show that if α is normal to base b then so is $n\alpha$ for any integer $n \geq 1$.

Exercise 3. Assume that α is irrational, and $d \geq 2$. Show that the sequence of vectors $\{\vec{v}_n = (\alpha n, \alpha n^2, \dots \alpha n^d) : n = 1, 2, \dots\} \subset \mathbb{R}^d$ is uniformly distributed on the torus $\mathbb{T}^d = \mathbb{R}^d/\mathbb{Z}^d$.

Hint: We know uniform distribution of $\alpha f(n)$ mod 1 where α is irrational and $f \in \mathbb{R}[x]$ is a non-constant monic polynomial.

Exercise 4. Let $f(x) = \alpha x^2 + \beta x + \gamma$, with $\alpha = p/q \neq 0$ rational, $p, q \in \mathbb{Z}\backslash\{0\}$, $q \geq 1$, β is irrational, and $\gamma \in \mathbb{R}$ is real. Show that $\{f(n) : n = 1, 2, \dots\}$ is uniformly distributed modulo one.

Hint: $f(qm + r) \equiv q\beta \cdot m + f(r) \bmod 1$ if $m, r \in \mathbb{Z}$.